

# ON THE FUNCTIONAL RENORMALIZATION GROUP APPROACH FOR YANG-MILLS FIELDS

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## Abstract

We explore the gauge dependence of the effective average action within the functional renormalization group (FRG) approach. It is shown that in the framework of standard definitions of FRG for the Yang-Mills theory, the effective average action remains gauge-dependent on-shell, independent on the use of truncation scheme. Furthermore, we propose a new formulation of the FRG, based on the use of composite operators. In this case one can provide on-shell gauge-invariance for the effective average action and universality of  $S$ -matrix.

**Keywords:** Renormalization group, Effective Average Action, Yang-Mills theories, Gauge dependence, BRST symmetry, Slavnov-Taylor identity, Composite fields.

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# 1 Introduction

The recent development of Quantum Field Theory is greatly related to the non-perturbative aspects of quantum theories. The request for such a non-perturbative treatment is related to the triviality problem of scalar field theory, non-perturbative nature of low-energy QCD and also an expectation to achieve a consistent theory of Quantum Gravity. One of the most promising approaches is related to different versions of Wilson renormalization group approach [1]. An important advance towards the use of the non-perturbative renormalization group has been done in the paper [2]. The qualitative idea of this work can be formulated as follows: regardless we do not know how to sum up the perturbative series, in some sense there is a good qualitative understanding of the final output of such a summation for the propagator of the quantum field. An exact propagator is supposed to have a single pole and also provide some smooth behavior in both UV and IR regions. It is possible to write a cut-off dependent propagator which satisfies these requirements. Then the cut-off dependence of the vertices can be established from the general scale-dependence of the theory which can be established by means of the functional methods. The method proved to be very helpful, in particular, for understanding the perturbative renormalization of the theory.

A compact and elegant formulation of the non-perturbative renormalization group has been proposed in [3, 4] in terms of effective action. The method was called functional renormalization group (we shall use abbreviation FRG) for the effective average action, it is nowadays one of the most popular and developed QFT methods, which can be seen from the review papers on the FRG method [5, 6, 7, 8, 9, 10].

As far as some of important applications of the FRG approach is related to QCD and Quantum Gravity, the special attention has been paid to the study of effective average action in gauge theories [11, 12] (see also [13, 14, 15, 16, 17, 18, 19, 20] and a very clear and complete review [21]). Many aspects of gauge theories in the framework of FRG has been discussed with success, but there is still one important question which remains unsolved. The consistent quantum description of gauge theories has to provide the on-shell independence on the choice of the gauge fixing condition. In a consistent formulation, such an independence should hold for the  $S$ -matrix elements and, equivalently, for the on-shell effective action. There is a good general understanding that this point represents a difficulty for the FRW approach [12, 14, 15, 16, 18, 20, 21], because the construction of FRG starts from the propagator, which is not a gauge invariant object and, in particular, always depends on the choice of gauge fixing condition. However, as far as we could see, the complete analysis of whether this general difficulty leads to problems at the level of  $S$ -matrix, was not done. The first purpose of the present work is to fill this gap, so we present a formal consideration of gauge dependence for the case of pure Yang-Mills theory. The method of analysis employed

here is based on a standard (albeit not really simple) use of BRST symmetry [22] which plays a fundamental role in Quantum Gauge Field Theory [23], and local form of Slavnov-Taylor identities [24, 25]. As we shall see in what follows, the regulator functions which emerge in the modified propagators do violate BRST symmetry and this leads, eventually, to the on-shell gauge dependence of the effective average action and, consequently, to the ambiguous  $S$ -matrix.

The existing attempts to solve the problem of gauge invariant formulation of FRG can be classified into two different types. The first one is based on reformulating the Yang-Mills theory with the help of a gauge-invariant cut-off-dependent regulator function introduced as a covariant form factor into the action of Yang-Mills fields, so that the regulated action is gauge invariant [26], [27] (see also earlier work [28]). Then the renormalization group equation is formulated. As far as there are no gauge fixing, one is free from the ambiguity related to the choice of the gauge condition. It is supposed that the infinite integral over the gauge group is absorbed into vacuum functional renormalization. It is not clear for us to which extent this approach for implementing covariant cut-off has relation to the effective average action of [11, 12]. An obvious deviation from the “canonical” method is that inserting the covariant form factor into the action of Yang-Mills fields means that the vertices also become cut-off dependent. According to [21], from the viewpoint of applications this, very interesting, approach requires dealing with complicated non-local structures. Also, the divergences which remain after integrating over the gauge group in the non-Abelian theory can depend on the Yang-Mills fields and, therefore, their removal without usual renormalisation procedure may be a difficult task.

The second approach [29, 30] is based on the use of Vilkovisky unique effective action [31] (see also [32] and [33] for further developments). The unique effective action provides gauge independence not only for the  $S$ -matrix, but even for the off-shell effective action. The price one has to pay is that this construction has its own ambiguities connected. It would be certainly interesting to have an alternative formulation of the effective average action, which would possess, in part of gauge dependence, the same properties as the conventional effective action in QFT. Namely, it may be gauge dependent off-shell, but should be gauge independent on-shell, such that the  $S$ -matrix would be unitary and well-defined.

As far as the source of the problem with gauge non-invariance is the introduction of the cut-off (or, better say, scale-dependent) propagator, it is clear that this is the aspect of the theory which should be reconsidered first. The known theorems about gauge-invariant renormalizability [34, 35] tell us that the exact effective action should be BRST-invariant. As far as the regulator functions which modify the propagator are supposed to mimic the all-loop quantum corrections, they must be taken in the BRST-invariant form. Therefore,

the problem is just to find the way to implement this invariance when one takes into account the regulator functions. The proposal which we present here is to use an old idea of [36] about introduction of composite operators in gauge theories. Following this line, we will introduce the regulator functions as composite operators and show that, in this case, the BRST symmetry is maintained at quantum level and, as a consequence, the  $S$ -matrix in the theory is gauge invariant and well defined.

The paper is organized as follows. The main features of the Faddeev-Popov method [37] for Yang-Mills fields are described in Section 2. For the pedagogical purposes we also include the demonstration of the gauge invariance of the vacuum functional and the on-shell invariance of effective action. Let us note that the rest of the paper relies on this important section in many respects, including definitions and notations. In Section 3, the FRG approach for Yang-Mills fields [4, 12] is briefly reviewed. In Section 4, the gauge dependence of vacuum functional in the FRG approach is investigated. In Section 5, the gauge dependence of effective average action of the FRG approach is explored. In Section 6 we present the new approach to the quantization of Yang-Mills fields with the regulator functions introduced by means of composite operators. Finally, Section 7 consists of concluding remarks and final discussions.

We use the standard condensed notation of DeWitt [38]. Derivatives with respect to sources and antifields are taken from the left, while those with respect to fields are taken from the right. Left derivatives with respect to fields are labeled by a subscript  $l$ . The Grassmann parity of a quantity  $F$  is denoted as  $\varepsilon(F)$ .

## 2 Yang-Mills theories within the Faddeev-Popov quantization

In this section we shall present some basic facts about Yang-Mills fields within the Faddeev-Popov quantization method [37]. Up to some extent, these considerations are general, but we restrict our attention to the Yang-Mills case only, just to stay within the scope of the present work. Our main purpose is to discuss the gauge independence of vacuum functional and, consequently, the gauge independence of the generating functional of the vertex functions (effective action) on-shell. Despite this material has not been presented earlier in exactly this form, the section has introductory purpose, and is intended to serve as a reference for the consequent consideration of the same issues in the framework of functional renormalization group approach to Yang-Mills theory which will be dealt with in the next sections.

The Yang-Mills fields  $A_\mu^a(x)$  belong to the adjoint representation of the  $SU(n)$  group, such that  $a = 1, \dots, n^2-1$ . The initial classical action  $S_0$  has the standard form,

$$S_0(A) = -\frac{1}{4} \int d^D x F_{\mu\nu}^a F^{\mu\nu a}, \quad \text{with} \quad F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f^{abc} A_\mu^b A_\nu^c, \quad (2.1)$$

where  $\mu, \nu = 0, 1, \dots, D-1$  and  $f^{abc}$  denote the (totally antisymmetric) structure constants of the Lie algebra  $SU(n)$ . We assume that Minkowski space has the signature  $(-, +, \dots, +)$ . The action (2.1) is invariant under gauge transformations

$$\delta A_\mu^a = D_\mu^{ab} \xi^b \quad \text{with} \quad D_\mu^{ab} = \delta^{ab} \partial_\mu + f^{acb} A_\mu^c \quad (2.2)$$

being generators of these transformations. After the Faddeev-Popov quantization, the field configuration space of Yang-Mills theory

$$\Phi^A = \{A_\mu^a, C^a, \bar{C}^a, B^a\}, \quad \text{with} \quad \varepsilon(C^a) = \varepsilon(\bar{C})^a = 1, \quad \varepsilon(A_\mu^a) = \varepsilon(B^a) = 0 \quad (2.3)$$

includes the (scalar) Faddeev-Popov ghost and antighost fields  $C^a$  and  $\bar{C}^a$ , respectively, as well as the Nakanishi-Lautrup auxiliary fields  $B^a$ . Choosing gauge fixing condition

$$\chi^a(A, B) = 0, \quad (2.4)$$

the Faddeev-Popov action,  $S_{FP}$ , is constructed in the form

$$S_{FP}(\Phi) = S_0(A) + \bar{C}^a M^{ab}(A, B) C^b + \chi^a(A, B) B^a, \quad (2.5)$$

with

$$M^{ab}(A, B) = \frac{\delta \chi^a(A, B)}{\delta A_\mu^c} D_\mu^{cb}. \quad (2.6)$$

The most popular gauge functions  $\chi^a$  in the Yang-Mills theory are the Landau gauge,

$$\chi^a = \partial^\mu A_\mu^a, \quad (2.7)$$

and the  $R_\xi$  gauge, defined by

$$\chi^a = \partial^\mu A_\mu^a + \frac{\xi}{2} B^a, \quad (2.8)$$

where  $\xi$  is an arbitrary gauge parameter. For these two cases the Faddeev-Popov matrices  $M^{ab}$  have the same form

$$M^{ab} = \partial^\mu D_\mu^{ab}. \quad (2.9)$$

The action (2.5) is invariant under the BRST transformation [22]

$$\delta_B A_\mu^a = D_\mu^{ab} C^b \theta, \quad \delta_B \bar{C}^a = B^a \theta, \quad \delta_B B^a = 0, \quad \delta_B C^a = \frac{1}{2} f^{abc} C^b C^c \theta, \quad (2.10)$$

where  $\theta$  is a constant Grassmann parameter. This transformation possesses a very important property of nilpotency. Let the BRST transformation be presented in the form

$$\delta_B \Phi^A = \hat{s} \Phi^A \theta, \quad \varepsilon(\Phi^A) = \varepsilon_A, \quad (2.11)$$

then one can verify the nilpotency of BRST transformation

$$\hat{s}^2 A_\mu^a = \hat{s} D_\mu^{ab} C^b = 0, \quad \hat{s}^2 \bar{C}^a = \hat{s} B^a = 0, \quad \hat{s}^2 B^a = 0, \quad \hat{s}^2 C^a = \hat{s} \frac{1}{2} f^{abc} C^b C^c = 0. \quad (2.12)$$

The generating functional of the Green's functions, which is sufficient to calculate all processes with Yang-Mills fields is given by the Faddeev-Popov formula [37]

$$Z(j) = \int \mathcal{D}\Phi \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi) + jA] \right\}, \quad (2.13)$$

where  $j = \{j_\mu^a(x)\}$  are sources of the fields  $A = \{A^{a\mu}(x)\}$ .

Thanks to the gauge invariance of the Yang-Mills action (2.1) and to the BRST invariance of the extended action (2.5), the Green's functions of the theory obey the relations known as the Slavnov- Taylor identities [24, 25]. These identities can be derived from (2.13) by means of the change of integration variables  $A_\mu^a$ , in the form of infinitesimal gauge transformations (2.2). The Jacobian of these transformations is equal to unity. Then the basic Slavnov-Taylor identities for Yang-Mills fields can be written in the form

$$j_\mu^a \langle D^{\mu ab} \rangle_j + \langle B^a \partial^\mu D_\mu^{ab} \rangle_j + f^{acb} \langle \bar{C}^a \partial^\mu D_\mu^{cd} C^d \rangle_j \equiv 0, \quad (2.14)$$

where the symbol  $\langle G \rangle_j$  means vacuum expectation value of the quantity  $G$  in the presence of external sources  $j_\mu^a$ ,

$$\langle G \rangle_j = \int \mathcal{D}\Phi G \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi) + jA] \right\}.$$

The generating functional  $Z(j)$  contains information about all Green's functions of the theory, which can be obtained by taking variational derivatives with respect to the sources. Similarly, the Slavnov-Taylor identities represent an infinite set of relations obtained from (2.14) by taking derivatives with respect to external sources  $j_\mu^a$ .

The form of the Slavnov-Taylor identities can be greatly simplified by introducing extra sources to the ghost, antighost and auxiliary fields. In this case one has to deal with the extended generating functional of the theory

$$Z(j, \bar{\eta}, \eta, \sigma) = \int \mathcal{D}\Phi \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi) + jA + \bar{\eta}C + \eta\bar{C} + \sigma B] \right\}. \quad (2.15)$$

It is clear that the relation to the conventional generating functional (2.13) performs as follows:

$$Z(j) = Z(j, \bar{\eta}, \eta, \sigma) \Big|_{\eta=\bar{\eta}=\sigma=0}. \quad (2.16)$$

The generating functional of connected Green's functions,  $W(j, \bar{\eta}, \eta, \sigma)$ , is defined by the relation

$$Z(j, \bar{\eta}, \eta, \sigma) = \exp \left\{ \frac{i}{\hbar} W(j, \bar{\eta}, \eta, \sigma) \right\}. \quad (2.17)$$

Finally, the generating functional of vertex Green's functions (effective action) is defined through the Legendre transformation of  $W$ ,

$$\Gamma(A, C, \bar{C}, B) = W(j, \bar{\eta}, \eta, \sigma) - jA - \bar{\eta}C - \eta\bar{C} - \sigma B, \quad (2.18)$$

where the source fields  $j, \bar{\eta}, \eta, \sigma$  are solutions of the equations

$$A^{a\mu}(x) = \frac{\delta W}{\delta j_\mu^a(x)}, \quad C^a(x) = \frac{\delta W}{\delta \bar{\eta}^a(x)}, \quad \bar{C}^a(x) = \frac{\delta W}{\delta \eta^a(x)}, \quad B^a(x) = \frac{\delta W}{\delta \sigma^a(x)}. \quad (2.19)$$

By means of (2.18) and (2.19) one can easily arrive at the relations

$$\begin{aligned} \frac{\delta \Gamma}{\delta A^{a\mu}(x)} &= -j_\mu^a(x), & \frac{\delta \Gamma}{\delta B^a(x)} &= -\sigma^a(x), \\ \frac{\delta \Gamma}{\delta C^a(x)} &= -\bar{\eta}^a(x), & \frac{\delta \Gamma}{\delta \bar{C}^a(x)} &= -\eta^a(x). \end{aligned} \quad (2.20)$$

The functional  $\Gamma$  satisfies the following functional integro-differential equation

$$\exp \left\{ \frac{i}{\hbar} \Gamma(\Phi) \right\} = \int \mathcal{D}\varphi \exp \left\{ \frac{i}{\hbar} \left[ S_{FP}(\Phi + \varphi) - \frac{\delta \Gamma(\Phi)}{\delta \Phi} \varphi \right] \right\}. \quad (2.21)$$

The last equation is a good starting point to perform the loop expansion, corresponding representation in form of the series in  $\hbar$ ,

$$\Gamma(\Phi) = \sum_{k=0}^{\infty} \hbar^k \Gamma^{(k)}(\Phi). \quad (2.22)$$

The solution can be immediately found in the tree approximation,

$$\Gamma^{(0)}(\Phi) = S_{FP}(\Phi).$$

The Slavnov-Taylor identities which are consequences of gauge symmetry of initial action can be rewritten with the help of BRST symmetry of the Faddeev-Popov action. For this end we make use of the change of variables in the functional integral (2.15) of the form (2.10). Because of the antisymmetry property of structure coefficients  $f^{abc}$  and nilpotency of  $\theta$ , the Jacobian of this transformation is equal to 1. Using the invariance of the functional integral under change of integration variables, the following identity holds

$$\int \mathcal{D}\Phi \left( j\delta_B A + \bar{\eta}\delta_B C + \eta\delta_B \bar{C} \right) \exp \left\{ \frac{i}{\hbar} \left[ S_{FP}(\Phi) + jA + \bar{\eta}C + \eta\bar{C} + \sigma B \right] \right\} \equiv 0. \quad (2.23)$$

Here the nilpotency of BRST transformation and the consequent exact relation

$$\exp \left\{ \frac{i}{\hbar} (j\delta_B A + \bar{\eta}\delta_B C + \eta\delta_B \bar{C}) \right\} = 1 + \frac{i}{\hbar} (j\delta_B A + \bar{\eta}\delta_B C + \eta\delta_B \bar{C}) \quad (2.24)$$

has been used.

Using the invariance (2.23), one can easily arrive at the Slavnov-Taylor identity for  $Z$ ,

$$j_\mu^a \partial^\mu \frac{\delta Z}{\delta \bar{\eta}^a} + \eta^a \frac{\delta Z}{\delta \sigma^a} + \frac{\hbar}{i} f^{acb} \left( J_\mu^a \frac{\delta^2 Z}{\delta j_\mu^c \delta \bar{\eta}^b} + \frac{1}{2} \bar{\eta}^a \frac{\delta^2 Z}{\delta \bar{\eta}^c \delta \bar{\eta}^b} \right) \equiv 0. \quad (2.25)$$

Due to the presence of the second-order variational derivatives of  $Z$ , the last identity has a non-local form. Fortunately, there exists a possibility to present the Slavnov-Taylor identity in the local form using the Zinn-Justin trick [39]. For the sake of symmetry and compactness of notations, we introduce the set of sources

$$J_A = (j_\mu^a, \bar{\eta}^a, \eta^a, \sigma^a), \quad \varepsilon(J_A) = \varepsilon(\Phi^A) = \varepsilon_A,$$

the set of external sources

$$K_A = (K_\mu^a, \bar{L}^a, L^a, N^a) \quad \varepsilon(K_A) = \varepsilon_A + 1$$

to the BRST transformation,  $\hat{s}\Phi^A$ , and the extended generating functional of Green's functions

$$\mathcal{Z}(J, K) = \int \mathcal{D}\Phi \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi) + J\Phi + K\hat{s}\Phi] \right\}, \quad (2.26)$$

where we used the notation for BRST transformations, which was previously introduced in (2.11). It is clear that

$$\mathcal{Z}(J, K) \Big|_{K=0} = Z(j, \bar{\eta}, \eta, \sigma). \quad (2.27)$$

Making use of the change of variables (2.10) in Eq. (2.27) and taking into account the nilpotency of BRST transformation (2.12), we obtain

$$\int \mathcal{D}\Phi J_A \hat{s}\Phi^A \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi) + J\Phi + K\hat{s}\Phi] \right\} \equiv 0 \quad (2.28)$$

or, equivalently,

$$J_A \frac{\delta \mathcal{Z}(J, K)}{\delta K_A} \equiv 0. \quad (2.29)$$

The last relation (2.29) represents the Slavnov-Taylor identity for Yang-Mills theory in the local form.



The Faddeev-Popov quantization of Yang-Mills theories provides a very important property of physical  $S$ -matrix being gauge independent. Let us discuss this aspect of the theory. As a first step, consider the vacuum functional  $\mathcal{Z}(0) \equiv \mathcal{Z}_\chi$  constructed for a given choice of gauge  $\chi^a = 0$ ,

$$\mathcal{Z}_\chi = \int \mathcal{D}\Phi \exp \left\{ \frac{i}{\hbar} S_{FP}(\Phi) \right\}. \quad (2.30)$$

Consider an infinitesimal change of the gauge fixing function  $\chi^a \rightarrow \chi^a + \delta\chi^a$ , corresponding to the new gauge fixing condition  $\chi^a + \delta\chi^a = 0$ . We have

$$\mathcal{Z}_{\chi+\delta\chi} = \int \mathcal{D}\Phi \exp \left\{ \frac{i}{\hbar} \left[ S_{FP}(\Phi) + \bar{C}^a \frac{\delta\delta\chi^a}{\delta A_\mu^c} D_\mu^{cb} C^b + \delta\chi^a B^a \right] \right\}. \quad (2.31)$$

Let us perform the change of variables according to Eqs. (2.10) in the functional integral (2.31), but with a functional  $\Lambda = \Lambda(\Phi)$  instead of the constant Grassmann odd variable  $\theta$ . Here  $\Lambda(\Phi)$  is supposed to be a Grassmann-odd quantity. Of course, the Faddeev-Popov action,  $S_{FP}$ , is invariant under such change of variables. The contributions come only from the integration measure, resulting in the corresponding Jacobian. Restricting our attention to the terms of the first order in the Grassmann-odd quantity  $\Lambda(\phi)$  and in the small quantity  $\delta\chi_\alpha(A)$ , one can rewrite the Jacobian according to the usual relation  $\text{sDet}(I + M) = \exp(\text{sTr}M)$ , where  $M^A_B \equiv \delta(\delta\Phi^A)/\delta\Phi^B$ . In this way we arrive at

$$\begin{aligned} \mathcal{Z}_{\chi+\delta\chi} = & \int \mathcal{D}\Phi \exp \left\{ \frac{i}{\hbar} \left[ S_{FP}(\Phi) + \bar{C}^a \frac{\delta\delta\chi^a}{\delta A_\mu^c} D_\mu^{cb} C^b + \delta\chi^a B^a \right. \right. \\ & \left. \left. + i\hbar \frac{\delta\Lambda}{\delta A_\mu^a} D_\mu^{ab} C^b - \frac{i\hbar}{2} f^{abc} C^b C^c \frac{\delta\Lambda}{\delta C^a} + i\hbar \frac{\delta\Lambda}{\delta \bar{C}^a} B^a \right] \right\}. \end{aligned} \quad (2.32)$$

By choosing the functional  $\Lambda(\phi)$  according to

$$\Lambda = \frac{i}{\hbar} \bar{C}^a \delta\chi^a,$$

it is easy to see from (2.32), that the vacuum functional does not depend on the choice of gauge, namely

$$\mathcal{Z}_{\chi+\delta\chi} = \mathcal{Z}_\chi. \quad (2.33)$$

Starting from this relation, one can prove the gauge independence of the  $S$  - matrix [40, 41].

The next part of our consideration concerns gauge independence on-shell for the effective action. We start by introducing the generalized generating functional of connected Green's functions

$$\mathcal{Z}(J, K) = \exp \left\{ \frac{i}{\hbar} \mathcal{W}(J, K) \right\}. \quad (2.34)$$

Now one can rewrite the Slavnov-Taylor identity (2.29) in terms of  $\mathcal{W}$  as

$$J_A \frac{\delta \mathcal{W}(J, K)}{\delta K_A} \equiv 0. \quad (2.35)$$

The effective action,  $\Gamma$ , is introduced through the Legendre transformation of  $\mathcal{W}$ ,

$$\Gamma(\Phi, K) = \mathcal{W}(J, K) - J_A \Phi^A, \quad \text{where} \quad \Phi^A = \frac{\delta \mathcal{W}}{\delta J_A}. \quad (2.36)$$

From (2.36) it follows that

$$\frac{\delta \Gamma}{\delta \Phi^A} = -J_A \quad \text{and} \quad \frac{\delta \Gamma}{\delta K_A} = \frac{\delta \mathcal{W}}{\delta K_A}. \quad (2.37)$$

By performing a shift of the integration variable, one can show that the functional  $\Gamma$  satisfies the equation

$$\exp \left\{ \frac{i}{\hbar} \Gamma(\Phi, K) \right\} = \int \mathcal{D}\varphi \exp \left\{ \frac{i}{\hbar} \left[ S_{FP}(\Phi + \varphi) - \frac{\delta \Gamma(\Phi, K)}{\delta \Phi} \varphi + K \hat{s}(\Phi + \varphi) \right] \right\}. \quad (2.38)$$

From (2.38) in tree approximation it follows

$$\Gamma^{(0)}(\Phi, K) = S_{FP}(\Phi) + K \hat{s}\Phi. \quad (2.39)$$

Now we are in a position to write down the Slavnov-Taylor identity in terms of  $\Gamma$ ,

$$\frac{\delta \Gamma}{\delta \Phi^A} \frac{\delta \Gamma}{\delta K_A} \equiv 0. \quad (2.40)$$

The effective action  $\Gamma$  is the main object of study in quantum theory of Yang-Mills fields, which contains all information about Green functions of the theory. By construction,  $\Gamma$  depends on gauge but this dependence has a very special form. Let us investigate this dependence. Consider infinitesimal variation of the gauge function  $\chi^a \rightarrow \chi^a + \delta\chi^a$  in the generating functional of the Green functions,  $\mathcal{Z} = \mathcal{Z}(J, K)$ ,

$$\begin{aligned} \delta \mathcal{Z} &= \frac{i}{\hbar} \int \mathcal{D}\Phi \left( \bar{C}^a \frac{\delta \delta \chi^a}{\delta A_\mu^c} D_\mu^{cb} C^b + B^a \delta \chi^a \right) \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi) + J\Phi + K \hat{s}\Phi] \right\} \\ &= \frac{i}{\hbar} \int \mathcal{D}\Phi \left[ \bar{C}^a \frac{\delta \delta \chi^a}{\delta A^{b\mu}} \frac{\delta(K \hat{s}\Phi)}{\delta K_\mu^b} + \delta \chi^a \frac{\delta(K \hat{s}\Phi)}{\delta L^a} \right] \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi) + J\Phi + K \hat{s}\Phi] \right\}. \end{aligned} \quad (2.41)$$

Introducing the functional

$$\delta\psi = \delta\psi(\Phi) = \bar{C}^a \delta\chi^a, \quad \varepsilon(\delta\psi) = 1 \quad (2.42)$$

we can rewrite (2.41) in the form

$$\delta \mathcal{Z} = \frac{i}{\hbar} \int \mathcal{D}\Phi \left[ \frac{\delta \delta\psi}{\delta A^{a\mu}} \frac{\delta(K \hat{s}\Phi)}{\delta K_\mu^a} + \frac{\delta \delta\psi}{\delta \bar{C}^a} \frac{\delta(K \hat{s}\Phi)}{\delta L^a} \right] \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi) + J\Phi + K \hat{s}\Phi] \right\}. \quad (2.43)$$

Taking into account that

$$\frac{\delta \delta \psi}{\delta C^a} = 0, \quad \frac{\delta(K \hat{s} \Phi)}{\delta N^a} = 0, \quad (2.44)$$

we can present the gauge dependence of generalized generating functional  $\mathcal{Z}$  (2.43) by the following relation

$$\delta \mathcal{Z} = \frac{i}{\hbar} \int \mathcal{D}\Phi \frac{\delta \delta \psi}{\delta \Phi^A} \frac{\delta(K \hat{s} \Phi)}{\delta K_A} \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi) + J\Phi + K \hat{s} \Phi] \right\}. \quad (2.45)$$

It is assumed that the functional integral of total variational derivative is zero. In this way we arrive at the relation

$$\int \mathcal{D}\Phi \frac{\delta}{\delta \Phi^A} \left[ \delta \psi \frac{\delta(K \hat{s} \Phi)}{\delta K_A} \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi) + J\Phi + K \hat{s} \Phi] \right\} \right] = 0. \quad (2.46)$$

After a small algebra it can be presented in the form

$$\begin{aligned} & \int \mathcal{D}\Phi \frac{\delta \delta \psi}{\delta \Phi^A} \frac{\delta(K \hat{s} \Phi)}{\delta K_A} \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi) + J\Phi + K \hat{s} \Phi] \right\} \\ &= -\frac{i}{\hbar} \int \mathcal{D}\Phi \delta \psi \left[ J_A \frac{\delta(K \hat{s} \Phi)}{\delta K_A} + \frac{\delta S_{FP}}{\delta \Phi^A} \frac{\delta(K \hat{s} \Phi)}{\delta K_A} + \frac{\delta(K \hat{s} \Phi)}{\delta \Phi^A} \frac{\delta(K \hat{s} \Phi)}{\delta K_A} \right. \\ & \quad \left. + \frac{\delta^2(K \hat{s} \Phi)}{\delta K_A \delta \Phi^A} \right] \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi) + J\Phi + K \hat{s} \Phi] \right\}. \end{aligned} \quad (2.47)$$

By means of the BRST symmetry of the Faddeev-Popov action

$$\frac{\delta S_{FP}}{\delta \Phi^A} \frac{\delta(K \hat{s} \Phi)}{\delta K_A} = \frac{\delta S_{FP}}{\delta \Phi^A} \hat{s} \Phi^A = 0, \quad (2.48)$$

using the nilpotency of the BRST transformations

$$\frac{\delta(K \hat{s} \Phi)}{\delta \Phi^A} \frac{\delta(K \hat{s} \Phi)}{\delta K_A} = K_B \hat{s} \delta_A^B \hat{s} \Phi^A = K_A \hat{s}^2 \Phi^A = 0 \quad (2.49)$$

and the equality

$$\frac{\delta^2(K \hat{s} \Phi)}{\delta K_A \delta \Phi^A} = \frac{\delta}{\delta K_A} \left[ \frac{\delta(K \hat{s} \Phi)}{\delta \Phi^A} \right] = \frac{\delta}{\delta K_A} K_A \hat{s} \cdot 1 = 0, \quad (2.50)$$

one can reduce the relation (2.47) to the form

$$\begin{aligned} & \int \mathcal{D}\Phi \frac{\delta \delta \psi}{\delta \Phi^A} \frac{\delta(K \hat{s} \Phi)}{\delta K_A} \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi) + J\Phi + K \hat{s} \Phi] \right\} \\ &= -\frac{i}{\hbar} \int \mathcal{D}\Phi \delta \psi(\Phi) J_A \frac{\delta(K \hat{s} \Phi)}{\delta K_A} \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi) + J\Phi + K \hat{s} \Phi] \right\}. \end{aligned} \quad (2.51)$$

With the help of (2.51), the gauge dependence of generalized generating functional  $\mathcal{Z}$  (2.45) can be rewritten in the form

$$\delta \mathcal{Z} = -\left(\frac{i}{\hbar}\right)^2 \int \mathcal{D}\Phi \delta \psi(\Phi) J_A \frac{\delta(K \hat{s} \Phi)}{\delta K_A} \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi) + J\Phi + K \hat{s} \Phi] \right\} \quad (2.52)$$

and furthermore as

$$\delta\mathcal{Z}(J, K) = \frac{i}{\hbar} J_A \frac{\delta}{\delta K_A} \delta\psi \left( \frac{\hbar}{i} \frac{\delta}{\delta J} \right) \mathcal{Z}(J, K). \quad (2.53)$$

This equation can be rewritten also in terms of the generating functional of connected Green's functions,  $\mathcal{W}$ , as

$$\delta\mathcal{W}(J, K) = J_A \left( \frac{i}{\hbar} \frac{\delta\mathcal{W}}{\delta K_A} + \frac{\delta}{\delta K_A} \right) \delta\psi \left( \frac{\delta\mathcal{W}}{\delta J} + \frac{\hbar}{i} \frac{\delta}{\delta J} \right). \quad (2.54)$$

Taken together with the Slavnov-Taylor identity for  $\mathcal{W}$  (2.35), the identity (2.54) can be presented as

$$\delta\mathcal{W}(J, K) = J_A \frac{\delta}{\delta K_A} \delta\psi \left( \frac{\delta\mathcal{W}}{\delta J} + \frac{\hbar}{i} \frac{\delta}{\delta J} \right). \quad (2.55)$$

Finally, after making Legendre transformation, one can arrive at the equation describing the gauge dependence of the effective action,

$$\delta\Gamma(\Phi, K) = - \frac{\delta\Gamma(\Phi, K)}{\delta\Phi^A} \frac{\delta}{\delta K_A} \delta\psi(\hat{\Phi}), \quad (2.56)$$

where the notation

$$\hat{\Phi}^A = \Phi^A + i\hbar (\Gamma''^{-1})^{AB} \frac{\delta_l}{\delta\Phi^B} \quad (2.57)$$

has been used. The matrix  $(\Gamma''^{-1})$  is inverse to the matrix  $\Gamma''$ , the last has elements

$$(\Gamma'')_{AB} = \frac{\delta_l}{\delta\Phi^A} \left( \frac{\delta\Gamma}{\delta\Phi^B} \right), \quad \text{i.e.,} \quad (\Gamma''^{-1})^{AC} \cdot (\Gamma'')_{CB} = \delta_B^A. \quad (2.58)$$

The main meaning of Eq. (2.56) is that the effective action does not depend on the choice of gauge function on-shell, which is defined by the effective equations of motion,

$$\frac{\delta\Gamma(\Phi, K)}{\delta\Phi^A} = 0 \quad \implies \quad \delta\Gamma(\Phi, K) = 0. \quad (2.59)$$

Of course, the same statement is valid for any physically relevant quantity, in particular the elements of the  $S$ -matrix are gauge independent in the same sense. This relevant feature can not be underestimated. Only due to the gauge independence one can interpret a calculated physical quantity as being independent on the method of calculation and, finally, consider the result being well defined. Let us stress that the relations (2.56) and (2.59) are not related to some approximation, such as, for instance, certain order of the loop expansion. Much on the contrary, those are very general non-perturbative relations, which must be provided in a well-defined quantum theory. The general property of the on-shell gauge independence can be, therefore, used as a natural test for a few method of deriving quantum corrections, in both perturbative and non-perturbative approaches.

### 3 Functional RG approach for Yang-Mills theories

In this section we are going to briefly present an approach to the calculation of effective action,  $\Gamma$ , proposed in paper [3, 11, 12] (for a review of the method see [21] and references therein), based on the concept of functional renormalization group (FRG). The main idea of the FRW is to use instead of  $\Gamma$  an effective average action,  $\Gamma_k$ , with a momentum-shell parameter  $k$ , such that

$$\lim_{k \rightarrow 0} \Gamma_k = \Gamma. \quad (3.1)$$

For the Yang-Mills theories it was suggested to modify the Faddeev-Popov action with the help of the specially designed regulator action  $S_k$

$$S_k(A, C, \bar{C}) = \int d^D x \left\{ \frac{1}{2} A^{a\mu}(x) (R_{k,A})_{\mu\nu}^{ab}(x) A^{b\nu}(x) + \bar{C}^a(x) (R_{k,gh})^{ab}(x) C^b(x) \right\}. \quad (3.2)$$

In what follows we will use the condensed notations,

$$S_k(A, C, \bar{C}) = \frac{1}{2} A^{a\mu} (R_{k,A})_{\mu\nu}^{ab} A^{b\nu} + \bar{C}^a (R_{k,gh})^{ab} C^b, \quad (3.3)$$

where regulator functions  $R_{k,A}$  and  $R_{k,gh}$  do not depend on the fields and obey the properties

$$\lim_{k \rightarrow 0} (R_{k,A})_{\mu\nu}^{ab} = 0, \quad \lim_{k \rightarrow 0} (R_{k,gh})^{ab} = 0.$$

It is assumed that the regulator functions model the non-perturbative contributions to the self-energy part of the diagrams, such that the dependence on the parameter  $k$  enables one to get some relevant information about the scale dependence of the theory beyond the loop expansion [2]. The application of the FRG method lead to many interesting achievements in many areas of Quantum Field Theory, Statistical Mechanics and related areas (see, e.g., the recent reviews [5, 6, 7, 8, 9, 10] and references therein).

Our immediate purpose is to check the consistency of the FRW method based on the introduction of (3.3), by exploring the gauge dependence of the effective average action, including the on-shell (in)dependence of this special version of effective action. As in the previous section, we shall restrict consideration by the pure Yang-Mills theory.

As a starting point, one has to note that the on-shell gauge independence which we have demonstrated in the previous section, is essentially dependent on the BRST invariance of the Faddeev-Popov action (2.5). Therefore, the first issue to check in the new FRG formulation is whether the BRST invariance holds in the presence of regulator functions. It is an easy exercise to verify that this is not the case, namely, the action (3.2) is not invariant under BRST transformations,

$$\delta_B S_k(A, C, \bar{C}) \neq 0. \quad (3.4)$$

Let us note that in the limit  $k \rightarrow 0$  the BRST invariance gets restored, but our interest is to use the concept of effective average action along the FRG trajectory and not only in its final point. In this case the output (3.4) should be seen as a warning signal, requesting a careful investigation of the issue of gauge dependence. The next step is to see whether this fact leads or not to the on-shell gauge dependence in this case. This task is much more complicated than the one described in section 2, hence we divide it between this and the next two sections.

The generating functional of Green's functions,  $\mathcal{Z}_k$ , is constructed in the form of the functional integral [12]

$$\mathcal{Z}_k(J, K) = \int \mathcal{D}\Phi \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi) + S_k(\Phi) + J\Phi + K\hat{s}\Phi] \right\}, \quad (3.5)$$

where, for the sake of uniformity, we used notation  $S_k(\Phi)$  instead  $S_k(A, C, \bar{C})$ , despite  $S_k$  does not depend on fields  $B^a$ . In the limit  $k \rightarrow 0$  this functional coincides with the generating functional (2.26).

The Slavnov-Taylor identity can be seen as a consequence of BRST invariance of the Faddeev-Popov action. In the case of the functional (3.5) this identity can be presented in the form

$$J_A \frac{\delta \mathcal{Z}_k}{\delta K_A} - i\hbar \left\{ (R_{k,A})_{\mu\nu}^{ab} \frac{\delta^2 \mathcal{Z}_k}{\delta j_\nu^b \delta K_\mu^a} + (R_{k,gh})^{ab} \frac{\delta^2 \mathcal{Z}_k}{\delta \eta^a \delta \bar{L}^b} - (R_{k,gh})^{ab} \frac{\delta^2 \mathcal{Z}_k}{\delta \bar{\eta}^b \delta L^a} \right\} \equiv 0. \quad (3.6)$$

In the limit  $k \rightarrow 0$  the last identity reduces to (2.29). In terms of generating functional of connected Green's functions,  $\mathcal{W}_k = \mathcal{W}_k(J, K)$ , the Slavnov-Taylor identity can be written in the form

$$\begin{aligned} & J_A \frac{\delta \mathcal{W}_k}{\delta K_A} + \left\{ (R_{k,A})_{\mu\nu}^{ab} \frac{\delta \mathcal{W}_k}{\delta j_\nu^b} \frac{\delta \mathcal{W}_k}{\delta K_\mu^a} + (R_{k,gh})^{ab} \frac{\delta \mathcal{W}_k}{\delta \eta^a} \frac{\delta \mathcal{W}_k}{\delta \bar{L}^b} - (R_{k,gh})^{ab} \frac{\delta \mathcal{W}_k}{\delta \bar{\eta}^b} \frac{\delta \mathcal{W}_k}{\delta L^a} \right\} \\ & - i\hbar \left\{ (R_{k,A})_{\mu\nu}^{ab} \frac{\delta^2 \mathcal{W}_k}{\delta j_\nu^b \delta K_\mu^a} + (R_{k,gh})^{ab} \frac{\delta^2 \mathcal{W}_k}{\delta \eta^a \delta \bar{L}^b} - (R_{k,gh})^{ab} \frac{\delta^2 \mathcal{W}_k}{\delta \bar{\eta}^b \delta L^a} \right\} \equiv 0. \end{aligned} \quad (3.7)$$

Finally, we introduce the generating functional of vertex functions in the presence of regulators (the effective average action),  $\Gamma_k = \Gamma_k(\Phi, K)$ , as

$$\Gamma_k(\Phi, K) = \mathcal{W}_k(J, K) - J\Phi, \quad \Phi^A = \frac{\delta \mathcal{W}_k}{\delta J_A}. \quad (3.8)$$

In the last expression the source  $J_A$  is regarded as a function of the mean field  $\Phi^A$ . Then

$$\frac{\delta \Gamma_k}{\delta \Phi^A} = -J_A, \quad \frac{\delta \Gamma_k}{\delta K_A} = \frac{\delta \mathcal{W}_k}{\delta K_A}. \quad (3.9)$$

The functional  $\Gamma_k$  satisfies the functional integro-differential equation

$$\begin{aligned} & \exp \left\{ \frac{i}{\hbar} \Gamma_k(\Phi, K) \right\} \\ &= \int \mathcal{D}\varphi \exp \left\{ \frac{i}{\hbar} \left[ S_{FP}(\Phi + \varphi) + S_k(\Phi + \varphi) + K \hat{s}(\Phi + \varphi) - \frac{\delta \Gamma_k(\Phi, K)}{\delta \Phi} \varphi \right] \right\}. \end{aligned} \quad (3.10)$$

The tree-level (zero-loop) approximation of (3.10) corresponds to

$$\Gamma_k^{(0)}(\Phi, K) = S_{FP}(\Phi) + S_k(\Phi) + K \hat{s}\Phi. \quad (3.11)$$

It proves useful to introduce another version of effective action, which does not depend on external sources  $K_A$ , such that the modified version of equation (3.10) gets simplified. For this end we define the functional  $\bar{\Gamma}_k$  according to

$$\bar{\Gamma}_k = \Gamma_k - K \hat{s}\Phi. \quad (3.12)$$

One can immediately find that  $\bar{\Gamma}_k$  does not depend on  $K$  and satisfies the equation

$$\exp \left\{ \frac{i}{\hbar} \bar{\Gamma}_k(\Phi) \right\} = \int \mathcal{D}\varphi \exp \left\{ \frac{i}{\hbar} \left[ S_{FP}(\Phi + \varphi) + S_k(\Phi + \varphi) - \frac{\delta \bar{\Gamma}_k(\Phi)}{\delta \Phi} \varphi \right] \right\}. \quad (3.13)$$

The derivation of the Slavnov-Taylor identity from the the BRST symmetry follows the same steps which we described in details in the previous section, so we can present just the final form of this identity in terms of  $\Gamma_k = \Gamma_k(\Phi, K)$ . The result reads

$$\begin{aligned} & \frac{\delta \Gamma_k}{\delta \Phi^A} \frac{\delta \Gamma_k}{\delta K_A} - \left\{ (R_{k,A})_{\mu\nu}^{ab} A^{b\nu} \frac{\delta \Gamma_k}{\delta K_\mu^a} + (R_{k,gh})^{ab} \bar{C}^a \frac{\delta \Gamma_k}{\delta \bar{L}^b} - (R_{k,gh})^{ab} C^b \frac{\delta \Gamma_k}{\delta L^a} \right\} \\ & - i\hbar \left\{ (R_{k,A})_{\mu\nu}^{ab} (\Gamma_k''^{-1})^{b\nu A} \frac{\delta_l^2 \Gamma_k}{\delta \Phi^A \delta K_\mu^a} + (R_{k,gh})^{ab} (\Gamma_k''^{-1})^{aA} \frac{\delta_l^2 \Gamma_k}{\delta \Phi^A \delta \bar{L}^b} \right. \\ & \left. - (R_{k,gh})^{ab} (\Gamma_k''^{-1})^{\bar{b}A} \frac{\delta_l^2 \Gamma_k}{\delta \Phi^A \delta L^a} \right\} \equiv 0. \end{aligned} \quad (3.14)$$

The matrix  $(\Gamma_k''^{-1})$  is inverse to the matrix  $\Gamma_k''$  with elements

$$(\Gamma_k'')_{AB} = \frac{\delta_l}{\delta \Phi^A} \left( \frac{\delta \Gamma_k}{\delta \Phi^B} \right), \quad \text{i.e.,} \quad (\Gamma_k''^{-1})^{AC} (\Gamma_k'')_{CB} = \delta_B^A. \quad (3.15)$$

In the last expressions we have used the following notation for indices:  $A = ((a\mu), a, \bar{a}, \tilde{a})$ , corresponding to the fields  $\Phi^A = (A^{a\mu}, C^a, \bar{C}^a, B^a)$ . In the zero-loop approximation,  $\Gamma_k(\Phi, K) = \Gamma_k^{(0)}(\Phi, K)$ , then the identity (3.14) reduces to the Zinn-Justin equation [39] for the action  $S_{ext}(\Phi, K) = S_{FP}(\Phi) + K \hat{s}\Phi$ , namely to

$$\frac{\delta S_{ext}}{\delta \Phi^A} \frac{\delta S_{ext}}{\delta K_A} = 0,$$

that corresponds to the BRST symmetry of the Faddeev-Popov action. Note that the Slavnov-Taylor identity in momentum space for the FRG approach was previously considered in the work [42].

The FRG flow equation written for the generating functional  $\mathcal{W}_k = \mathcal{W}_k(J, K)$  can be written in detailed form as follows:

$$\begin{aligned} \partial_t \mathcal{W}_k &= \frac{1}{2} \partial_t (R_{k,A})_{\mu\nu}^{\text{ab}} \frac{\delta \mathcal{W}_k}{\delta j_\mu^a} \frac{\delta \mathcal{W}_k}{\delta j_\nu^b} + \partial_t (R_{k,gh})^{\text{ab}} \frac{\delta \mathcal{W}_k}{\delta \eta^a} \frac{\delta \mathcal{W}_k}{\delta \bar{\eta}^b} \\ &- i\hbar \left\{ \frac{1}{2} \partial_t (R_{k,A})_{\mu\nu}^{\text{ab}} \frac{\delta^2 \mathcal{W}_k}{\delta j_\mu^a \delta j_\nu^b} + \partial_t (R_{k,gh})^{\text{ab}} \frac{\delta^2 \mathcal{W}_k}{\delta \eta^a \delta \bar{\eta}^b} \right\}, \end{aligned} \quad (3.16)$$

where we used a standard notation

$$\partial_t = k \frac{d}{dk}$$

and took into account that the dependence on  $k$  comes only from the corresponding dependence of the regulator functions (3.2).

Consider the FRG flow equation for the effective average action,  $\Gamma_k$ . From the definition of  $\Phi$  in (3.8) one can see that it is dependent on the parameter  $k$ , that means  $\partial_t \Phi^A \neq 0$ . Therefore, one has to be very careful in calculations and take into account all ways of  $k$ -dependence. We have

$$\partial_t \Gamma_k \Big|_{\Phi, K} + \frac{\delta \Gamma_k}{\delta \Phi^A} \Big|_{k, K} \partial_t \Phi^A = \partial_t \mathcal{W}_k \Big|_{J, K} - J_A \partial_t \Phi^A \Big|_{J, K}, \quad (3.17)$$

where the index  $\Big|_X$  after partial or variational derivative means that the quantity  $X$  is kept constant. Due to the properties of the Legendre transformations (3.9), we obtain

$$\partial_t \Gamma_k \Big|_{\Phi, K} = \partial_t \mathcal{W}_k \Big|_{J, K}. \quad (3.18)$$

As far as the FRG parameter  $k$  is not physical, we can not expect that it will emerge in the final physical output of the theory. Therefore, summing up all types of  $k$ -dependence we always get zero. The application of the renormalization group method implies that we take into account only part of  $k$ -dependence, e.g., find how the effective action depends on  $k$  and then trade it for some physical parameter corresponding to the problem of our interest (see detailed discussion of this issue in [43]). Therefore, in the FRG flow equation for  $\Gamma_k$ , only the explicit dependence on  $k$  should be taken into account, so we get

$$\partial_t \Gamma_k = \partial_t S_k + i\hbar \left\{ \frac{1}{2} \partial_t (R_{k,A})_{\mu\nu}^{\text{ab}} (\Gamma_k^{\prime\prime-1})^{(a\mu)(b\nu)} + \partial_t (R_{k,gh})^{\text{ab}} (\Gamma_k^{\prime\prime-1})^{\text{ab}} \right\}. \quad (3.19)$$

Usually the functional RG approach is formulated in terms of the functional which does not depend on sources  $K$ . Since the equation (3.19) does not contain derivatives with respect to  $K$ , we can just put  $K_A = 0$  and arrive at

$$\partial_t \bar{\Gamma}_k = \partial_t S_k + i\hbar \left\{ \frac{1}{2} \partial_t (R_{k,A})_{\mu\nu}^{\text{ab}} (\bar{\Gamma}_k^{\prime\prime-1})^{(a\mu)(b\nu)} + \partial_t (R_{k,gh})^{\text{ab}} (\bar{\Gamma}_k^{\prime\prime-1})^{\text{ab}} \right\}, \quad (3.20)$$



where

$$\bar{\Gamma}_k = \bar{\Gamma}_k(\Phi) = \Gamma_k(\Phi, K = 0).$$

$\bar{\Gamma}_k(\Phi)$  satisfies equation (3.13). In the condensed notations we can write down the equation (3.20) in the form

$$\partial_t \bar{\Gamma}_k = \partial_t S_k + i\hbar \left\{ \frac{1}{2} \text{Tr} [\partial_t (R_{k,A}) (\bar{\Gamma}_k''^{-1})]_A - \text{Tr} [\partial_t (R_{k,gh}) (\bar{\Gamma}_k''^{-1})]_C \right\}, \quad (3.21)$$

where we took into account the anticommuting nature of the ghost fields  $C^A$  and defined

$$\begin{aligned} \text{Tr} [\partial_t (R_{k,gh}) (\bar{\Gamma}_k''^{-1})]_C &= -\partial_t (R_{k,gh})^{ab} (\bar{\Gamma}_k''^{-1})^{ab}, \\ \text{Tr} [\partial_t (R_{k,A}) (\bar{\Gamma}_k''^{-1})]_A &= \partial_t (R_{k,A})^{ab} (\bar{\Gamma}_k''^{-1})^{(a\mu)(b\nu)}. \end{aligned} \quad (3.22)$$

In the tree-level approximation we have  $\bar{\Gamma}_k = S_{FP} + S_k$  and the equation (3.21) is satisfied because the Faddeev-Popov action does not depend on  $k$ . The conventional presentation of the FRG flow equation is in terms of the functional  $\Gamma_k = \bar{\Gamma}_k - S_k$ , which satisfies the equation

$$\partial_t \Gamma_k = i\hbar \left\{ \frac{1}{2} \text{Tr} [\partial_t R_{k,A} (\Gamma_k'' + R_{k,A})^{-1}]_A - \text{Tr} [\partial_t (R_{k,gh}) (\Gamma_k'' + R_{k,gh})^{-1}]_C \right\}. \quad (3.23)$$

The last representation uses symbolic notations of inverse matrices  $(\Gamma_k'' + R_{k,A})^{-1}$  and  $(\Gamma_k'' + R_{k,gh})^{-1}$ , which are in fact components of the following inverse matrix

$$(\Gamma_k + S_k)''_{AB} = \frac{\delta_l}{\delta \Phi^A} \left[ \frac{\delta (\Gamma_k + S_k)}{\delta \Phi^B} \right] \quad (3.24)$$

in the sectors of vector and ghost fields, respectively.

## 4 Vacuum functional in the FRG for Yang-Mills field

Let us explore the vacuum functional in the FRG approach. We introduce the notation  $Z_{k,\chi}$  for the vacuum functional, corresponding to the given gauge function  $\chi^a$  in the Faddeev-Popov action

$$Z_{k,\chi} = \mathcal{Z}_k(0,0) = \int \mathcal{D}\Phi \exp \left\{ \frac{i}{\hbar} (S_{FP} + S_k) \right\}. \quad (4.1)$$

By construction, the regulator functions in the FRG approach do not depend on gauge  $\chi^a$  and therefore the action  $S_k$  is gauge independent. Let us consider an infinitesimal variation of gauge  $\chi \rightarrow \chi + \delta\chi$  and construct the vacuum functional corresponding to this gauge. We have

$$Z_{k,\chi+\delta\chi} = \int \mathcal{D}\Phi \exp \left\{ \frac{i}{\hbar} \left( S_{FP} + S_k + \bar{C}^a \frac{\delta \chi^a}{\delta A_\mu^c} D_\mu^{cb} C^b + \delta \chi^a B^a \right) \right\}. \quad (4.2)$$

In the functional integral (4.2) we make a change of variables in the form of the BRST transformations (2.10), but trading the constant Grassmann-odd parameter  $\theta$  to a functional  $\Lambda = \Lambda(\Phi)$ . The Faddeev-Popov action,  $S_{FP}$ , is invariant under such change of variables but  $S_k$  is not invariant, with the variation given by

$$\delta S_k = A^{a\mu} (R_{k,A})_{\mu\nu}^{ab} D^{\nu bc} C^c \Lambda + \frac{1}{2} \bar{C}^a (R_{k,gh})^{ab} f^{bcd} C^c C^d \Lambda - B^a (R_{k,gh})^{ab} C^b \Lambda. \quad (4.3)$$

The contributions which come from the measure of functional integral have the form<sup>1</sup>

$$i\hbar \left( \frac{\delta \Lambda}{\delta A_\mu^a} D_\mu^{ab} C^b - \frac{1}{2} f^{abc} C^b C^c \frac{\delta \Lambda}{\delta C^a} + \frac{\delta \Lambda}{\delta \bar{C}^a} B^a \right). \quad (4.4)$$

Let us discuss the possibility to compensate the variation (4.3) by choosing a special form of the functional  $\Lambda$ . The transformed value of the generating functional can be obtained by combining the formulas (4.2), (4.3) and (4.4),

$$\begin{aligned} Z_{k,\chi+\delta\chi} &= \int \mathcal{D}\Phi \exp \left\{ \frac{i}{\hbar} \left[ S_{FP}(\Phi) + S_k + \bar{C}^a \frac{\delta \delta \chi^a}{\delta A_\mu^c} D_\mu^{cb} C^b - B^a (R_{k,gh})^{ab} C^b \Lambda \right. \right. \\ &\quad + \frac{1}{2} \bar{C}^a (R_{k,gh})^{ab} f^{bcd} C^c C^d \Lambda + A^{a\mu} (R_{k,A})_{\mu\nu}^{ab} D^{\nu bc} C^c \Lambda + \delta \chi^a B^a \\ &\quad \left. \left. - \frac{i\hbar}{2} f^{abc} C^b C^c \frac{\delta \Lambda}{\delta C^a} + i\hbar \frac{\delta \Lambda}{\delta A_\mu^a} D_\mu^{ab} C^b + i\hbar \frac{\delta \Lambda}{\delta \bar{C}^a} B^a \right] \right\}. \end{aligned} \quad (4.5)$$

In order to provide compensation of all new terms in  $Z_{k,\chi+\delta\chi}$ , such that it becomes equal to  $Z_{k,\chi}$ , one has to satisfy the following equations:

$$\begin{aligned} i\hbar \frac{\delta \Lambda}{\delta A_\mu^a} - A^{b\nu} (R_{k,A})_{\nu\mu}^{ba} \Lambda + \bar{C}^b \frac{\delta \delta \chi^b}{\delta A_\mu^a} &= 0, \\ i\hbar \frac{\delta \Lambda}{\delta \bar{C}^a} + \delta \chi^a - (R_{k,gh})^{ab} C^b \Lambda &= 0, \end{aligned} \quad (4.6)$$

$$i\hbar \frac{\delta \Lambda}{\delta C^a} + \frac{1}{2} \bar{C}^b (R_{k,gh})^{ba} \Lambda = 0. \quad (4.7)$$

It is easy to see that the solution of the last equation,

$$\frac{\delta \Lambda}{\delta C^a} = \frac{1}{2i\hbar} \bar{C}^b (R_{k,gh})^{ba} \Lambda, \quad (4.8)$$

has the form

$$\Lambda = \frac{1}{2i\hbar} \bar{C}^a (R_{k,gh})^{ab} C^b + \Lambda_1(\bar{C}, B, A). \quad (4.9)$$

At the same time, the first of the equations (4.7) can be cast into the form

$$i\hbar \frac{\delta \Lambda_1}{\delta A_\mu^a} + \bar{C}^b \frac{\delta \delta \chi^b}{\delta A_\mu^a} - A^{b\nu} (R_{k,A})_{\nu\mu}^{ba} \Lambda_1 - \frac{1}{2i\hbar} A^{b\nu} (R_{k,A})_{\nu\mu}^{ba} \bar{C}^d (R_{k,gh})^{dc} C^c = 0 \quad (4.10)$$

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<sup>1</sup>Compare to the considerations leading to Eq. (2.32).

and its solution should be dependent on the field  $C^a$  as well. One can conclude that this contradicts the (4.9) and, therefore, it is not possible to compensate the variation (4.3) by choosing a special form of the functional  $\Lambda$ .

For instance, if we choose  $\Lambda$  in a natural way,

$$\Lambda = \frac{i}{\hbar} \bar{C}^a \delta\chi^a,$$

then

$$Z_{k,\chi+\delta\chi} = \int \mathcal{D}\Phi \exp \left\{ \frac{i}{\hbar} (S_{FP} + S_k + \delta S_k) \right\}. \quad (4.11)$$

Then, for any value  $k \neq 0$ , one has

$$Z_{k,\chi+\delta\chi} \neq Z_{k,\chi}. \quad (4.12)$$

Therefore the vacuum functional  $Z_{k,\chi}$  (and therefore  $S$ -matrix) depends on gauge.

One might think that situation with gauge dependence within the FRG approach can be improved if we propose gauge dependence of regulators  $(R_{k,A})_{\mu\nu}^{ab}$  and  $(R_{k,gh})^{ab}$ , such that the gauge variation of  $S_k$  gives additional contributions

$$\frac{1}{2} A^{a\mu} \delta(R_{k,A})_{\mu\nu}^{ab} A^{b\nu} + \bar{C}^a \delta(R_{k,gh})^{ab} C^b. \quad (4.13)$$

Unfortunately, one can easily verify that the terms appearing in the exponent inside the functional integral cannot be compensated by choosing the functional  $\Lambda$ . Therefore, the possible generalization (4.13) can not solve the problem of gauge dependence of vacuum functional within the FRW approach.

## 5 Gauge dependence of $\Gamma_k$

Let us explore the gauge dependence of the generating functionals  $\mathcal{Z}_k$ ,  $\mathcal{W}_k$  and  $\Gamma_k$  for Yang-Mills theory in the framework of the FRG approach. We shall follow the general methods from the papers [34, 44]. The derivation of this dependence is based on a variation of the gauge-fixing function,  $\chi^a \rightarrow \chi^a + \delta\chi^a$ , which leads to the variation of the Faddeev-Popov action  $S_{FP}$  (2.5) and consequently of the generating functional  $\mathcal{Z}_k = \mathcal{Z}_k(J, K)$  (3.5). Introducing the functional  $\delta\psi = \bar{C}^a \delta\chi^a$ , the gauge dependence can be presented in the form

$$\delta\mathcal{Z}_k = \frac{i}{\hbar} \int \mathcal{D}\Phi \frac{\delta\delta\psi}{\delta\Phi^A} \frac{\delta(K\hat{s}\Phi)}{\delta K_A} \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi) + S_k + J\Phi + K\hat{s}\Phi] \right\}. \quad (5.1)$$

Let us consider an obvious relation

$$\int \mathcal{D}\Phi \frac{\delta}{\delta\Phi^A} \left\{ \delta\psi \frac{\delta(K\hat{s}\Phi)}{\delta K_A} \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi) + S_k + J\Phi + K\hat{s}\Phi] \right\} \right\} = 0. \quad (5.2)$$

It is not difficult to obtain its extended form,

$$\begin{aligned} & \int \mathcal{D}\Phi \frac{\delta \delta \psi}{\delta \Phi^A} \frac{\delta(K \hat{s} \Phi)}{\delta K_A} \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi) + S_k + J\Phi + K \hat{s} \Phi] \right\} \\ &= -\frac{i}{\hbar} \int \mathcal{D}\Phi \delta \psi(\Phi) \left( J_A + \frac{\delta S_k}{\delta \Phi^A} \right) \frac{\delta(K \hat{s} \Phi)}{\delta K_A} \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi) + S_k + J\Phi + K \hat{s} \Phi] \right\}. \end{aligned} \quad (5.3)$$

Using the last formula, we can rewrite gauge dependence of the functional  $\mathcal{Z}_k$  in the form

$$\delta \mathcal{Z}_k = -\left(\frac{i}{\hbar}\right)^2 \int \mathcal{D}\Phi \delta \psi(\Phi) \left( J_A + \frac{\delta S_k}{\delta \Phi^A} \right) \frac{\delta(K \hat{s} \Phi)}{\delta K_A} \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi) + S_k + J\Phi + K \hat{s} \Phi] \right\}.$$

Taking into account the explicit structure of  $S_k$  (3.3) one can get the relation

$$\begin{aligned} \frac{\delta S_k}{\delta \Phi^A} \frac{\delta(K \hat{s} \Phi)}{\delta K_A} &= A^{a\mu}(R_{k,A})_{\mu\nu}^{\text{ab}} \frac{\delta(K \hat{s} \Phi)}{\delta j_\nu^b} \\ &+ \bar{C}^a(R_{k,gh})^{\text{ab}} \frac{\delta(K \hat{s} \Phi)}{\delta \bar{L}^b} - C^a(R_{k,gh})^{ba} \frac{\delta(K \hat{s} \Phi)}{\delta L^b}. \end{aligned} \quad (5.4)$$

For the final form of gauge dependence of generating functional  $\mathcal{Z}_k = \mathcal{Z}_k(J, K)$  we obtain the relation

$$\begin{aligned} \delta \mathcal{Z}_k &= \frac{i}{\hbar} J_A \frac{\delta}{\delta K_A} \delta \psi \left( \frac{\hbar}{i} \frac{\delta}{\delta J} \right) \mathcal{Z}_k + \\ &+ \left[ (R_{k,A})_{\mu\nu}^{\text{ab}} \frac{\delta^2}{\delta j_\mu^a \delta j_\nu^b} + (R_{k,gh})^{\text{ab}} \frac{\delta^2}{\delta \eta^a \delta \bar{L}^b} - (R_{k,gh})^{ba} \frac{\delta^2}{\delta \bar{\eta}^a \delta L^b} \right] \delta \psi \left( \frac{\hbar}{i} \frac{\delta}{\delta J} \right) \mathcal{Z}_k. \end{aligned} \quad (5.5)$$

The last equation can be rewritten in terms of the generating functional of the connected Green's functions,  $\mathcal{W}_k = \mathcal{W}_k(J, K)$ , to give

$$\begin{aligned} \delta \mathcal{W}_k &= J_A \frac{\delta}{\delta K_A} \delta \psi \left( \frac{\delta \mathcal{W}_k}{\delta J} + \frac{\hbar}{i} \frac{\delta}{\delta J} \right) - i\hbar \left[ (R_{k,A})_{\mu\nu}^{\text{ab}} \frac{\delta^2}{\delta j_\mu^a \delta j_\nu^b} \right. \\ &+ \left. (R_{k,gh})^{\text{ab}} \frac{\delta^2}{\delta \eta^a \delta \bar{L}^b} - (R_{k,gh})^{ba} \frac{\delta^2}{\delta \bar{\eta}^a \delta L^b} \right] \delta \psi \left( \frac{\delta \mathcal{W}_k}{\delta J} + \frac{\hbar}{i} \frac{\delta}{\delta J} \right), \end{aligned} \quad (5.6)$$

where the identity (3.7) was used.

The next step is to write the corresponding equation for the effective average action. It proves convenient to introduce the following notation:

$$S_{k;A}(\Phi) = \frac{\delta S_k(\Phi)}{\delta \Phi^A}. \quad (5.7)$$

Then the equation (5.6) reads

$$\delta \mathcal{W}_k = \left\{ J_A - i\hbar S_{k;A} \left( \frac{\delta}{\delta J} \right) \right\} \frac{\delta}{\delta K_A} \delta \psi \left( \frac{\delta \mathcal{W}_k}{\delta J} + \frac{\hbar}{i} \frac{\delta}{\delta J} \right). \quad (5.8)$$

In terms of the effective average action this equation becomes

$$\delta\Gamma_k = -\frac{\delta\Gamma_k}{\delta\Phi^A} \frac{\delta}{\delta K_A} \delta\psi(\hat{\Phi}) - i\hbar S_{k;A} \left( \frac{i}{\hbar} (\Phi - \hat{\Phi}) \right) \frac{\delta}{\delta K_A} \delta\psi(\hat{\Phi}), \quad (5.9)$$

where  $\hat{\Phi}^A$  is defined as

$$\hat{\Phi}^A = \Phi^A + i\hbar (\Gamma_k''^{-1})^{AB} \frac{\delta_l}{\delta\Phi^B}. \quad (5.10)$$

The inverse matrix  $(\Gamma_k''^{-1})^{AB}$  was introduced in (3.15). We see that if on-shell is defined in the usual way,

$$\frac{\delta\Gamma_k}{\delta\Phi^A} = 0, \quad (5.11)$$

then even on-shell the effective average action  $\Gamma_k$  depends on gauge,  $\delta\Gamma_k \neq 0$ . This result confirms the one of the previous section and shows that the gauge dependence represents a serious problem for the FRG approach in the standard conventional formulation.

## 6 An alternative approach with composite operators

In this section we are going to suggest an approach in spirit of the FRG, which is free of the gauge dependence problem for Yang-Mills theory. This new approach is based on implementing regulator functions by means of composite fields.

Effective action for composite fields in Quantum Field Theory was introduced in [45]. Later on, effective action for composite fields in gauge theories was introduced and studied in the papers [36, 46, 47]. In the case of gauge theories this effective action depends on gauge. It was shown that this dependence has a very special form and that there is a possibility to define a theory with composite fields in such a way that the effective action of these fields becomes gauge independent on-shell.

Let us see how the composite fields idea can be used in the FRG framework for gauge theories. The idea is to use such a fields to implement regulator functions. Consider the regulator functions

$$L_k^1(x) = \frac{1}{2} A^{a\mu}(x) (R_{k,A})_{\mu\nu}^{ab}(x) A^{b\nu}(x), \quad (6.1)$$

$$L_k^2(x) = \bar{C}^a(x) (R_{k,gh})^{ab}(x) C^b(x). \quad (6.2)$$

Now we introduce external scalar sources  $\Sigma_1(x)$  and  $\Sigma_2(x)$  and construct the generating functional of Green's functions for Yang-Mills theories with composite fields

$$\mathcal{Z}_k(J, K; \Sigma) = \int \mathcal{D}\Phi \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi) + J\Phi + K\hat{s}\Phi + \Sigma L_k(\Phi)] \right\}. \quad (6.3)$$

where  $\Sigma L_k = \Sigma_1 L_k^1 + \Sigma_2 L_k^2$ .

The generating functional (6.3) may be regarded as a generalization of the generating functionals discussed previously, the difference is related to the new sources  $\Sigma_1(x)$  and  $\Sigma_2(x)$  and to the corresponding composite fields. By choosing the sources  $\Sigma$  to have zero values,  $\Sigma_1(x) = \Sigma_2(x) = 0$ , the functional (6.3) boils down to the generating functional for Yang-Mills theories described in Sect. 2. At the same time, when  $\Sigma_1(x) = \Sigma_2(x) = 1$ , the functional (6.3) corresponds to the generating functional in the FRG approach introduced in Sect. 3.

One can note that at least one advantage of the proposal (6.3) is quite evident from the very beginning. If we define the vacuum functional where all the sources are switched off,  $J = K = \Sigma = 0$ , then it coincides with the vacuum functional for Yang-Mills theory and hence does not depend on the gauge fixing. Let us now see how this feature concerns vacuum functional and effective action.

Consider the FRG flow equations in the approach with composite operators, based on the new generating functional (6.3). Starting from (6.3) one can repeat the considerations of Sect. 3 and arrive at the new version of the FRG flow equation for the generating functional  $\mathcal{Z}_k = \mathcal{Z}_k(J, K; \Sigma)$ ,

$$\partial_t \mathcal{Z}_k = \frac{\hbar}{i} \left\{ \frac{1}{2} \Sigma_1 \partial_t (R_{k,A})_{\mu\nu}^{ab} \frac{\delta^2 \mathcal{Z}_k}{\delta j_\mu^a \delta j_\nu^b} + \Sigma_2 \partial_t (R_{k,gh})^{ab} \frac{\delta^2 \mathcal{Z}_k}{\delta \eta^a \delta \bar{\eta}^b} \right\}. \quad (6.4)$$

As usual, we should rewrite it in terms of  $\mathcal{W}_k = \mathcal{W}_k(J, K; \Sigma)$ ,

$$\begin{aligned} \partial_t \mathcal{W}_k &= \frac{1}{2} \Sigma_1 \partial_t (R_{k,A})_{\mu\nu}^{ab} \frac{\delta \mathcal{W}_k}{\delta j_\mu^a} \frac{\delta \mathcal{W}_k}{\delta j_\nu^b} + \Sigma_2 \partial_t (R_{k,gh})^{ab} \frac{\delta \mathcal{W}_k}{\delta \eta^a} \frac{\delta \mathcal{W}_k}{\delta \bar{\eta}^b} \\ &- i\hbar \left\{ \frac{1}{2} \Sigma_1 \partial_t (R_{k,A})_{\mu\nu}^{ab} \frac{\delta^2 \mathcal{W}_k}{\delta j_\mu^a \delta j_\nu^b} + \Sigma_2 \partial_t (R_{k,gh})^{ab} \frac{\delta^2 \mathcal{W}_k}{\delta \eta^a \delta \bar{\eta}^b} \right\}. \end{aligned} \quad (6.5)$$

The effective average action with composite fields,  $\Gamma_k = \Gamma_k(\Phi, K; F)$ , can be introduced by means of the following double Legendre transformations (see [45] for the details in case of usual effective action):

$$\Gamma_k(\Phi, K; F) = \mathcal{W}_k(J, K; \Sigma) - J_A \Phi^A - \Sigma_i [L_k^i(\Phi) + \hbar F^i], \quad (6.6)$$

where

$$\Phi^A = \frac{\delta \mathcal{W}_k}{\delta J_A}, \quad \hbar F^i = \frac{\delta \mathcal{W}_k}{\delta \Sigma_i} - L_k^i \left( \frac{\delta \mathcal{W}_k}{\delta J} \right), \quad i = 1, 2. \quad (6.7)$$

From (6.6) and (6.7) follows that

$$\frac{\delta \Gamma_k}{\delta \Phi^A} = -J_A - \Sigma_i \frac{\delta L_k^i(\Phi)}{\delta \Phi^A}, \quad \frac{\delta \Gamma_k}{\delta F^i} = -\hbar \Sigma_i. \quad (6.8)$$

Let us introduce the full sets of fields  $\mathcal{F}^{\mathcal{A}}$  and sources  $\mathcal{J}_{\mathcal{A}}$  according to

$$\mathcal{F}^{\mathcal{A}} = (\Phi^A, \hbar F^i), \quad \mathcal{J}_{\mathcal{A}} = (J_A, \hbar \Sigma_i). \quad (6.9)$$

From the condition of solvability of equations (6.8) with respect to the sources  $J$  and  $\Sigma$ , it follows that

$$\frac{\delta \mathcal{F}^{\mathcal{C}}(\mathcal{J})}{\delta \mathcal{J}_{\mathcal{B}}} \frac{\delta_l \mathcal{J}_{\mathcal{A}}(\mathcal{F})}{\delta \mathcal{F}^{\mathcal{C}}} = \delta_{\mathcal{A}}^{\mathcal{B}}. \quad (6.10)$$

One can express  $\mathcal{J}_{\mathcal{A}}$  as a function of the fields in the form

$$\mathcal{J}_{\mathcal{A}} = \left( -\frac{\delta \Gamma_k}{\delta \Phi^A} - \frac{\delta \Gamma_k}{\delta F^i} \frac{\delta L_k^i(\Phi)}{\delta \Phi^A}, -\frac{\delta \Gamma_k}{\delta F^i} \right) \quad (6.11)$$

and, therefore,

$$\frac{\delta_l \mathcal{J}_{\mathcal{B}}(\mathcal{F})}{\delta \mathcal{F}^{\mathcal{A}}} = -(G_k'')_{\mathcal{A}\mathcal{B}}, \quad \frac{\delta \mathcal{F}^{\mathcal{B}}(\mathcal{J})}{\delta \mathcal{J}_{\mathcal{A}}} = -(G_k''^{-1})^{\mathcal{A}\mathcal{B}}. \quad (6.12)$$

Now we are in a position to derive the FRW equation for  $\Gamma_k = \Gamma_k(\Phi, K; F)$  and see that it gains more complicated form due to the presence of composite fields. The expression for the generating functional of connected Green's functions can be obtained from (6.3),

$$\exp \left\{ \frac{i}{\hbar} \mathcal{W}_k(J, K; \Sigma) \right\} = \int \mathcal{D}\Phi' \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi') + J\Phi' + K\hat{s}\Phi' + \Sigma L_k(\Phi')] \right\}. \quad (6.13)$$

Taking into account Eq. (6.6), the equation (6.13) in terms of  $\Gamma_k$  reads

$$\begin{aligned} & \exp \left\{ \frac{i}{\hbar} \left[ \Gamma_k(\Phi, K; F) + J\Phi + \Sigma(L_k(\Phi) + \hbar F) \right] \right\} \\ &= \int \mathcal{D}\Phi' \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi') + J\Phi' + K\hat{s}\Phi' + \Sigma L_k(\Phi')] \right\}. \end{aligned}$$

An equivalent form of this equation is

$$\begin{aligned} & \exp \left\{ \frac{i}{\hbar} \left[ \Gamma_k(\Phi, K; F) + \hbar \Sigma F \right] \right\} \\ &= \int \mathcal{D}\Phi' \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi') + J(\Phi' - \Phi) + K\hat{s}\Phi' + \Sigma(L_k(\Phi') - L_k(\Phi))] \right\}. \end{aligned}$$

Making shift of the variables of integration in the functional integral,  $\Phi' - \Phi = \varphi$ , and using (6.8), we obtain the equation for the effective action  $\Gamma_k = \Gamma_k(\Phi, K; F)$ ,

$$\begin{aligned} \exp \left\{ \frac{i}{\hbar} \left[ \Gamma_k - \frac{\delta \Gamma_k}{\delta F} F \right] \right\} &= \int \mathcal{D}\varphi \exp \left\{ \frac{i}{\hbar} \left[ S_{FP}(\Phi + \varphi) - \frac{\delta \Gamma_k}{\delta \Phi} \varphi \right. \right. \\ &\quad \left. \left. + K\hat{s}(\Phi + \varphi) - \frac{1}{\hbar} \frac{\delta \Gamma_k}{\delta F} \left( L_k(\Phi + \varphi) - L_k(\Phi) - \frac{\delta L_k(\Phi)}{\delta \Phi} \varphi \right) \right] \right\}. \end{aligned} \quad (6.14)$$

The solution of this equation in the tree-level approximation has the form

$$\Gamma_k^{(0)}(\Phi, K; F) = S_{FP}(\Phi) + K\hat{s}\Phi, \quad (6.15)$$

which does not depend on the fields  $F^i$  and parameter  $k$ . The next step is to define loop corrections to (6.15), so we assume that

$$\Gamma_k(\Phi, K; F) = S_{FP}(\Phi) + K\hat{s}\Phi + \hbar\bar{\Gamma}_k(\Phi; F). \quad (6.16)$$

By taking into account the explicit structure of regulator functions we obtain the equation which can be, for example, a basis for deriving the loop expansion of  $\bar{\Gamma}_k = \bar{\Gamma}_k(\Phi; F)$ ,

$$\begin{aligned} \exp \left\{ i \left[ \bar{\Gamma}_k - \frac{\delta \bar{\Gamma}_k}{\delta F} F \right] \right\} &= \int \mathcal{D}\varphi \exp \left\{ \frac{i}{\hbar} \left[ S_{FP}(\Phi + \varphi) - S_{FP}(\Phi) - \frac{\delta S_{FP}(\Phi)}{\delta \Phi} \varphi \right. \right. \\ &\quad \left. \left. - \hbar \frac{\delta \bar{\Gamma}_k}{\delta \Phi} \varphi - \frac{\delta \bar{\Gamma}_k}{\delta F} L_k(\varphi) \right] \right\}. \end{aligned} \quad (6.17)$$

Using the properties of the Legendre transformation, one can arrive at the relation

$$\partial_t \mathcal{W}_k = \partial_t \Gamma_k - \frac{1}{\hbar} \frac{\delta \Gamma_k}{\delta F^i} \partial_t L_k^i(\Phi), \quad (6.18)$$

where the derivatives  $\partial_t$  are calculated with respect to the explicit dependence of  $\Gamma_k$ ,  $\mathcal{W}_k$  and  $L_k^i(\Phi)$  on the regulator parameter  $k$ .

Finally, the FRG flow equation, in terms of the functional  $\Gamma_k$ , is cast into the form

$$\begin{aligned} \partial_t \Gamma_k &= -i \left\{ \frac{1}{2} \frac{\delta \Gamma_k}{\delta F^1} \partial_t (R_{k,A})_{\mu\nu}^{ab} (G_k''^{-1})^{(a\mu)(b\nu)} + \frac{\delta \Gamma_k}{\delta F^2} \partial_t (R_{k,gh}) (G_k''^{-1})^{ab} \right\} \\ &= -\frac{i}{2} \text{Tr} \left\{ \frac{\delta \Gamma_k}{\delta F^1} \partial_t (R_{k,A}) (G_k''^{-1}) \right\}_A + i \text{Tr} \left\{ \frac{\delta \Gamma_k}{\delta F^2} \partial_t (R_{k,gh}) (G_k''^{-1}) \right\}_C, \end{aligned} \quad (6.19)$$

when we used usual traces in the sectors of vector  $A^{a\mu}$  and ghost  $C^a$  fields while the Grassmann parity of quantum fields is taken into account explicitly.

Consider now a consequence of the BRST invariance of  $S_{FP}$  for the generating functional (6.3). Making use of the change of variables (2.10) in (6.3) and taking into account the nilpotency of BRST transformation (2.12), we arrive at the identity

$$\begin{aligned} \int \mathcal{D}\Phi \left\{ J_A \hat{s} \Phi^A + \Sigma_1 (R_{k,A})_{\mu\nu}^{ab} A^{b\nu} \hat{s} A^{a\mu} + \Sigma_2 \bar{C}^a (R_{k,gh})^{ab} \hat{s} C^a \right. \\ \left. - \Sigma_2 (R_{k,gh})^{ab} C^b \hat{s} \bar{C}^a \right\} \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi) + J\Phi + K\hat{s}\Phi + \Sigma L_k] \right\} \equiv 0. \end{aligned} \quad (6.20)$$

The same equation can be written in terms of the generating functional (6.3),

$$J_A \frac{\delta \mathcal{Z}_k}{\delta K_A} + \frac{\hbar}{i} \left\{ \Sigma_1 (R_{k,A})_{\mu\nu}^{ab} \frac{\delta^2 \mathcal{Z}_k}{\delta j_\nu^b \delta K_\mu^a} + \Sigma_2 (R_{k,gh})^{ab} \frac{\delta^2 \mathcal{Z}_k}{\delta \eta^a \delta \bar{L}^a} - \Sigma_2 (R_{k,gh})^{ab} \frac{\delta^2 \mathcal{Z}_k}{\delta \bar{\eta}^b \delta L^a} \right\} \equiv 0. \quad (6.21)$$



The identity (6.21) can be considered as the generalized Slavnov-Taylor identity in the presence of composite fields. In the limits of  $\Sigma_1(x) = \Sigma_2(x) = 0$  and  $\Sigma_1(x) = \Sigma_2(x) = 1$  this identity coincides with the previously considered identities (2.29) and (3.6), respectively.

Let us explore the gauge dependence of generating functional of Green's functions in the presence of composite fields, (6.3). As before, one can consider an infinitesimal variation of gauge function,  $\chi^a \rightarrow \chi^a + \delta\chi^a$ . Taking into account the results obtained in Sect. 2, we arrive at the following variation of  $\mathcal{Z}_k = \mathcal{Z}_k(J, K; \Sigma)$ :

$$\delta\mathcal{Z}_k = \frac{i}{\hbar} \int \mathcal{D}\Phi \frac{\delta\delta\psi}{\delta\Phi^A} \frac{\delta(K\hat{s}\Phi)}{\delta K_A} \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi) + J\Phi + K\hat{s}\Phi + \Sigma L_k] \right\}, \quad (6.22)$$

where the Grassmann-odd functional  $\delta\psi = \bar{C}^a \delta\chi^a$  has been introduced. Starting from the identity

$$\int \mathcal{D}\Phi \frac{\delta}{\delta\Phi^A} \left[ \delta\psi \frac{\delta(K\hat{s}\Phi)}{\delta K_A} \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi) + J\Phi + K\hat{s}\Phi + \Sigma L_k] \right\} \right] = 0, \quad (6.23)$$

one can derive the relation

$$\begin{aligned} & \int \mathcal{D}\Phi \frac{\delta\delta\psi}{\delta\Phi^A} \frac{\delta(K\hat{s}\Phi)}{\delta K_A} \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi) + J\Phi + K\hat{s}\Phi + \Sigma L_k] \right\} \\ &= -\frac{i}{\hbar} \int \mathcal{D}\Phi \delta\psi \left( J_A + \Sigma \frac{\delta L_k}{\delta\Phi^A} \right) \frac{\delta(K\hat{s}\Phi)}{\delta K_A} \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi) + J\Phi + K\hat{s}\Phi + \Sigma L_k] \right\}. \end{aligned} \quad (6.24)$$

Using the last formula together with Eq. (6.22), one can show that the gauge dependence of  $\mathcal{Z}_k$  is described by the equation

$$\begin{aligned} \delta\mathcal{Z}_k &= -\left(\frac{i}{\hbar}\right)^2 \int \mathcal{D}\Phi \delta\psi \left( J_A + \Sigma \frac{\delta L_k}{\delta\Phi^A} \right) \frac{\delta(K\hat{s}\Phi)}{\delta K_A} \times \\ &\times \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi) + J\Phi + K\hat{s}\Phi + \Sigma L_k] \right\}. \end{aligned} \quad (6.25)$$

Taking into account the explicit structure of regulator functions  $L_k^i(\Phi)$  in Eqs. (6.1) and (6.2), one can rewrite the second term in the integrand of (6.25) as

$$\begin{aligned} \Sigma \frac{\delta L_k}{\delta\Phi^A} \frac{\delta(K\hat{s}\Phi)}{\delta K_A} &= \Sigma_1 A^{a\mu} (R_{k,A})_{\mu\nu}^{ab} \frac{\delta(K\hat{s}\Phi)}{\delta j_\nu^b} \\ &+ \Sigma_2 \bar{C}^a (R_{k,gh})^{ab} \frac{\delta(K\hat{s}\Phi)}{\delta \bar{L}^b} - \Sigma_2 C^a (R_{k,gh})^{ba} \frac{\delta(K\hat{s}\Phi)}{\delta L^b}. \end{aligned} \quad (6.26)$$

Using this relation, we obtain the final equation, describing the gauge dependence of the generating functional  $\mathcal{Z}_k = \mathcal{Z}_k(J, K; \Sigma)$ ,

$$\begin{aligned} \delta\mathcal{Z}_k &= \frac{i}{\hbar} J_A \frac{\delta}{\delta K_A} \delta\psi \left( \frac{\hbar}{i} \frac{\delta}{\delta J} \right) \mathcal{Z}_k + \left\{ \Sigma_1 (R_{k,A})_{\mu\nu}^{ab} \frac{\delta^2 \mathcal{Z}_k}{\delta j_\mu^a \delta j_\nu^b} \right. \\ &+ \left. \Sigma_2 (R_{k,gh})^{ab} \frac{\delta^2 \mathcal{Z}_k}{\delta \eta^a \delta \bar{L}^b} - \Sigma_2 (R_{k,gh})^{ba} \frac{\delta^2 \mathcal{Z}_k}{\delta \bar{\eta}^a \delta L^b} \right\} \delta\psi \left( \frac{\hbar}{i} \frac{\delta}{\delta J} \right) \mathcal{Z}_k. \end{aligned} \quad (6.27)$$

In the limits  $\Sigma_1(x) = \Sigma_2(x) = 0$  and  $\Sigma_1(x) = \Sigma_2(x) = 1$ , this equation coincides with Eqs. (2.53) and (5.5), respectively.

In terms of generating functional of connected Green's functions,  $\mathcal{W}_k = \mathcal{W}_k(J, K)$ , the equation (6.27) can be written as

$$\begin{aligned} \delta\mathcal{W}_k = & \mathcal{J}_A \frac{\delta}{\delta K_A} \delta\psi \left( \frac{\delta\mathcal{W}_k}{\delta J} + \frac{\hbar}{i} \frac{\delta}{\delta J} \right) - i\hbar \left\{ \Sigma_1(R_{k,A})_{\mu\nu}^{ab} \frac{\delta^2}{\delta j_\mu^a \delta j_\nu^b} \right. \\ & \left. + \Sigma_2(R_{k,gh})^{ab} \frac{\delta^2}{\delta \eta^a \delta \bar{L}^b} - \Sigma_2(R_{k,gh})^{ba} \frac{\delta^2}{\delta \bar{\eta}^a \delta L^b} \right\} \delta\psi \left( \frac{\delta\mathcal{W}_k}{\delta J} + \frac{\hbar}{i} \frac{\delta}{\delta J} \right), \end{aligned} \quad (6.28)$$

where the identity (3.7) was used. If we introduce the notations

$$L_{k,A}^i(\Phi) = \frac{\partial L_k^i(\Phi)}{\partial \Phi^A}, \quad i = 1, 2, \quad (6.29)$$

the equation (6.28) can be cast in the compact form,

$$\delta\mathcal{W}_k = \left\{ J_A - i\hbar \Sigma_i L_{k,A}^i \left( \frac{\hbar}{i} \frac{\delta}{\delta J} \right) \right\} \frac{\delta}{\delta K_A} \delta\psi \left( \frac{\delta\mathcal{W}_k}{\delta J} + \frac{\hbar}{i} \frac{\delta}{\delta J} \right). \quad (6.30)$$

Finally, the gauge dependence of the effective average action follows from (6.30) after one performs the Legendre transform. It is described by the equation

$$\delta\Gamma_k = - \frac{\delta\Gamma_k}{\delta\Phi^A} \frac{\delta}{\delta K_A} \delta\psi(\hat{\Phi}) - \frac{1}{\hbar} \frac{\delta\Gamma_k}{\delta F^i} L_{k,A}^i(\Phi - \hat{\Phi}) \frac{\delta}{\delta K_A} \delta\psi(\hat{\Phi}). \quad (6.31)$$

Let us define the mass-shell of the quantum theory by the equations

$$\frac{\delta\Gamma_k}{\delta\Phi^A} = 0, \quad \frac{\delta\Gamma_k}{\delta F^i} = 0. \quad (6.32)$$

Then from (6.31) immediately follows the gauge independence of the effective action  $\Gamma_k = \Gamma_k(\Phi, K; F)$  on-shell. Namely, when the relations (6.32) are satisfied, we have  $\delta\Gamma_k = 0$ . Moreover, all physical quantities calculated on the basis of the modified version of the effective average action do not depend on gauge and on the parameter  $k$ . The last feature is common for the renormalization group based on the abstract scale parameters (such as cut-off,  $k$  or  $\mu$ ), which require an additional identification of scale to be applied to one or another physical problem. For example, we know that the  $S$ -matrix elements in usual Quantum Field Theory do not depend on  $\mu$  in the Minimal Subtraction scheme of renormalization. This does not mean that there is no running, of course (see [43] for detailed discussion of this issue). In our case, after the evaluation of  $\Gamma_k$  is completed in a given approximation (including, perhaps, the truncation scheme) one has to identify  $k$  with some physical quantity and only after that go on-shell and calculate physical quantities, such as  $S$ -matrix elements<sup>2</sup>.

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<sup>2</sup>In most complicated cases, such as Quantum (or semiclassical) Gravity, when direct comparison with the physical renormalization scheme (like momentum subtraction) is not possible, the scale-setting is tricky, but still admits a regular procedure which works well in different physical situations [48, 49].

## 7 Conclusions and final discussions

We investigated, in much more details than it was done before, the problem of gauge dependence in the functional renormalization group (FRG) approach. The consideration was performed for the generating functional of the Green functions and effective action, but the main target was the universality of the definition of  $S$ -matrix and, more general, on-shell gauge dependence of the effective action.

The regulator functions which are introduced in the FRG formalism to model the behavior of exact Green's functions lead to the breakdown of the BRST symmetry. As a result, the effective average action depends on the choice of gauge fixing even on-shell. The situation is qualitatively similar to the Gribov-Zwanziger theory [50, 51, 52], where the restrictions on the domain of integration in functional integral, which is due to the Gribov horizon, violates the BRST symmetry and consequently leads to on-shell gauge dependence of the effective action of the theory [53, 54]. As another example of this sort we can mention the situation with the theory possessing global supersymmetry (modified BRST symmetry) [55]. When the nilpotency of the global supersymmetry is violated, one meets a problem of gauge dependence for the relevant physical quantities.

One can consider the situation with gauge dependence of  $S$ -matrix and on-shell effective action in two different ways. The first one implies that we consider the effective average action as an approximation to the real effective action in a given theory. As far as the last is gauge independent on-shell and leads to the well-defined physical predictions, one may think that the effective average action produces an approximation to the invariant quantities in a given gauge, which should be taken as granted. This kind of consideration is, in some sense, the unique option if we do not invent an alternative gauge-invariant formulation of the FRG approach. However, it is obvious that the second way, that is constructing a gauge-invariant formulation, would mean a much better approximation.

As a first step forward in formulating the gauge-invariant version of the FRG, we propose the new formulation of the theory with cut-off dependent regulator functions. It was shown that if these functions are introduced by means of the special composite fields, the BRST symmetry is preserved at quantum level, vacuum functional does not depend on the choice of gauge and, finally, the new theory is free from the on-shell gauge dependence. It would be very interesting to compare this new approach to the standard one. For example, it looks interesting to verify the gauge independence for the approximate higher-loop  $\beta$ -function of the theory in the new and traditional approaches. The derivation of such a  $\beta$ -function and, in general, the loop expansion within the FRG theory with composite fields, lies beyond the scope of the present work. Indeed, we expect to deal with these and other related subjects in the next publications.

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